A Statistical Fusion for a Leg-by-leg Bearings-Only TMA without Observer Maneuver

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Abstract - In previous papers, we addressed the problem of bearings-only tracking of targets whose trajectory is composed of several legs from a non-maneuvering observer. We named it bearings-only maneuvering target motion analysis (BOMTMA). We proposed two estimates when the number of legs is equal to two: the classic maximum likelihood estimate (MLE) and an estimate based on a geometric fusion of leg-by-leg tracking. In this paper we introduce an estimate constructed on a statistical fusion whose main advantage, unlike the two previous ones, is that it can be easily extended to the case where the trajectory of the source is composed of three or more legs.

Keywords: Maneuvering target, bearings-only target motion analysis, TMA, BOTMA, BOMTMA, maximum likelihood estimate, Cramèr-Rao lower bound, fusion.

1 Introduction

The bearings-only target motion analysis (BOTMA) consists of estimating the trajectory of a source whose velocity is constant during the period of measurement from a set of bearings collected by an observer; the two mobiles are supposed moving in the same plane [1]. In order to guarantee observability [2] [3] and to obtain an accurate estimate, the observer must maneuver efficiently [4] [5]. This problem is essentially met in the submarine context. In two recent papers [6] [7], we proved that, conversely, if the observer has a constant velocity and the trajectory of the source is composed of two legs at constant speed (see Fig.1), then, under a simple condition on the velocities, the source is observable. For this problem, called bearings-only maneuvering target motion analysis (BOMTMA), we first proposed the maximum likelihood estimate (MLE) and showed that this estimate is relatively efficient. The disadvantage is that the observer must wait for the maneuver of the source and detect it. From an operational point of view, this can be unacceptable whereas the observer could obtain a piece of information from the collected bearings during the first leg. With this in mind, assuming that maneuver time is known (or finely estimated), we proposed a "partial" BOTMA approach of what is observable during the first leg, then during the second one, and then fusing these two estimated state vectors we obtained an estimate of the source trajectory (see [8]). The way of fusing, based upon the constant speed of the source and called "geometric fusion", is still questionable for essentially two main reasons:

- We do not exploit the covariance matrices of the estimates given by the two partial BOTMA.
- The estimated trajectory can be discontinuous.

We propose here a new way to fuse the results of the two or more partial BOTMA which alleviates these drawbacks.

This paper is composed of four main sections:

- In section 2, we recall the context of the classic BOTMA.
- Section 3 is devoted to a short reminder of the partial BOTMA.
- In section 4, after having summarized the geometric fusion, we present the new approach called statistical fusion.
- Finally, some results on simulated data are presented in section 5.

2 Problem formulation of the conventional BOTMA

The classic hypotheses of the BOTMA are the following [1]: a source S and a passive observer O (also called own ship) are moving in the same plane according to their own velocity. During the observation, the source has a constant velocity. At time t, relative to a Cartesian coordinate system, the location $P_S(t) = [x_S(t) \quad y_S(t)]^T$ of the source and its velocity $V_S = [\dot{x}_S \quad \dot{y}_S]^T$ obey the following motion equation

$$\begin{cases} x_S(t) = x_S(t^*) + (t - t^*)\dot{x}_S \\ y_S(t) = y_S(t^*) + (t - t^*)\dot{y}_S \end{cases} \quad (1)$$

where t^* is an arbitrary reference time. As a consequence, such a motion is entirely defined by the state vector $X_S = [x_S(t^*) \quad y_S(t^*) \quad \dot{x}_S \quad \dot{y}_S]^T$.

It is well known [2] [3] [10] that the state vector X_S is observable by an observer located (at time t) at $P_O(t) = [x_O(t) \quad y_O(t)]^T$ collecting a set of bearings

$$\left\{ \theta(t, X_S) = \operatorname{atan}\left[\frac{x_S(t) - x_O(t)}{y_S(t) - y_O(t)}\right], \text{ for } t_1 \le t \le t_K \right\},$$

provided the observer maneuvers properly during $[t_1, t_K]$. In the rest of the paper, we are interested by what the observer can do when it does not maneuver during the time of observation. We will study cases where the trajectory of the source is composed of one leg, two legs and finally three legs.

3 One leg: the partial BOTMA

In this context, the source keeps a constant velocity $V_S = [\dot{x}_S \quad \dot{y}_S]^T$. It is straightforward to check [7] that, if $\theta(t)$ is not constant (assumption made from now on), the set of homothetic trajectories producing the same noise-free-bearings from the observer is

$$\Lambda = \{ X(\lambda) = \lambda(X_S - X_O) + X_O, \ for \ \lambda > 0 \} \quad (3)$$

with $X_O = [x_O(t^*) \quad y_O(t^*) \quad \dot{x}_O \quad \dot{y}_O]^T$,

$V_O = [\dot{x}_O \quad \dot{y}_O]^T$ being the velocity of the observer.

The previous set Λ can be characterized by any of its elements. Indeed, if X^* is a particular element of Λ, then the set $\Lambda^* = \{ X(\mu) = \mu(X^* - X_O) + X_O, \ for \ \mu > 0 \}$ is equal to Λ.

If $\lambda = \dfrac{x_{fix} - x_O(t^*)}{x_S(t^*) - x_O(t^*)}$, for some value x_{fix}, such as $\lambda > 0$, then the components of $X(\lambda)$ are $[x_{fix} \quad \lambda(y_S - y_O) + y_O \quad \lambda(\dot{x}_S - \dot{x}_O) + \dot{x}_O \quad \lambda(\dot{y}_S - \dot{y}_O) + \dot{y}_O]^T$. The condition $\lambda > 0$ is satisfied when $x_{fix} = \bar{\rho}\sin\theta(t^*, X_S) + x_O(t^*)$, with $\bar{\rho} > 0$. So, $\lambda = \dfrac{\bar{\rho}\sin\theta(t^*, X_S)}{x_S(t^*) - x_O(t^*)}$ (*). For the sake of simplicity, we will write X instead of $X(\lambda)$ and its components will be denoted $[x_{fix} \quad y \quad \dot{x} \quad \dot{y}]^T$. This will help us in sub-section 4.1.

At time t_k, the observer measures the azimuth of the line of sight in which the source is detected:

$$\beta_k = \theta(t_k, X_S) + \varepsilon_k, \ k = 1, \cdots, K$$

where ε_k is assumed to be a zero-mean Gaussian noise of variance σ_k^2 (assumed known).

With no loss of generality, we choose $t^* = t_K$. The chosen state vector is $X = [x_{fix} \quad y \quad \dot{x} \quad \dot{y}]^T$, with $x_{fix} = \bar{\rho}\sin\beta_K + x_O(t_K)$. Note that the quantity $\bar{\rho}$ is arbitrarily chosen. Only the last three components have to be estimated. Assuming the covariance matrix of the random vector $(\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_K)^T$ to be diagonal, the log-likelihood function to maximize is proportional to the least squares criterion:

$$C(X) = \sum_{k=1}^{K} \frac{1}{\sigma_k^2}[\beta_k - \theta(t_k, X)]^2$$

The Gauss-Newton method [11] used for this minimization, initialized at

$$X_{init} = \begin{bmatrix} \bar{\rho}\sin\beta_K + x_O(t_K) \\ \bar{\rho}\cos\beta_K + y_O(t_K) \\ \dfrac{\bar{\rho}\sin\beta_K + x_O(t_K) - \bar{\rho}\sin\beta_1 - x_O(t_1)}{t_K - t_1} \\ \dfrac{\bar{\rho}\cos\beta_K + x_O(t_K) - \bar{\rho}\cos\beta_1 - x_O(t_1)}{t_K - t_1} \end{bmatrix}.$$

returns the maximum likelihood estimate (MLE), denoted $\hat{X} = [\bar{\rho}\sin\beta_K + x_O(t_K) \quad \hat{y} \quad \hat{\dot{x}} \quad \hat{\dot{y}}]^T$. Of course, $\hat{y}$, $\hat{\dot{x}}$ and $\hat{\dot{y}}$ depend on the choice of $\bar{\rho}$ (hence on the corresponding but unknown λ).

4 Two legs: the BOMTMA by a leg-by-leg approach

Now, the trajectory of the source is composed of two legs at constant speed: during the first leg which starts at t_1 and finishes at time t_M (assumed known), the velocity of the source is denoted $V_{S,1} = [\dot{x}_{S,1} \quad \dot{y}_{S,1}]^T$; the second leg starts at t_M and finishes at t_K and the velocity is $V_{S,2} = [\dot{x}_{S,2} \quad \dot{y}_{S,2}]^T$, satisfying the assumption that $\|V_{S,1}\| = \|V_{S,2}\| = v_S$. The whole trajectory is hence characterized by the state vector $Z = [x_S(t_M) \quad y_S(t_M) \quad v_S \quad h_{S,1} \quad h_{S,2}]^T$ (coordinates of position at time t_M, speed, courses of the first and of second leg). Meanwhile, the observer keeps a constant speed vector $V_O = [\dot{x}_O \quad \dot{y}_O]^T$.

The observability condition of Z, *i.e.* $V_O^T(V_{S,1} - V_{S,2}) \neq 0$, is satisfied from now on [6].

We define the vector

$$X_{S,i} = [x_{S,i}(t_M) \quad y_{S,i}(t_M) \quad \dot{x}_{S,i} \quad \dot{y}_{S,i}]^T,$$

relative to leg #i.. Note that $y_{S,1}(t_M) = y_{S,2}(t_M)$, denoted in the sequel y_S.

4.1 The geometric fusion

The principle of the geometric fusion is articulated by two steps [8]:

(*) a convenient rotation helps to avoid the case where $x_S(t^*) - x_O(t^*)$ =0.

1) Step 1 (partial BOTMA - one for each leg - with $t^* = t_M$):

We define for leg #1 the state vector $X_1 = [\bar{\rho}\sin\beta_M + x_O(t_M) \quad y_1 \quad \dot{x}_1 \quad \dot{y}_1]^T$ and for leg #2 the state vector $X_2 = [\bar{\rho}\sin\beta_M + x_O(t_M) \quad y_2 \quad \dot{x}_2 \quad \dot{y}_2]^T$. Note that the first two components of X_1 and X_2 are equal : $y_1 = y_2 = y$.

The partial BOTMA provides us the estimate

$$\hat{X}_1 = \left[\bar{\rho}\sin\beta_M + x_O(t_M) \quad \hat{y}_1 \quad \hat{\dot{x}}_1 \quad \hat{\dot{y}}_1\right]^T$$

of X_1 for leg #1 and similarly the estimate

$$\hat{X}_2 = \left[\bar{\rho}\sin\beta_M + x_O(t_M) \quad \hat{y}_2 \quad \hat{\dot{x}}_2 \quad \hat{\dot{y}}_2\right]^T$$

of X_2 for leg #2.

2) Step 2 (geometric fusion)

We compute from $\hat{X}_1$ and $\hat{X}_2$ the homothetic estimates, $\mu(\hat{X}_1 - X_O) + X_O$ for the first leg and $\mu(\hat{X}_2 - X_O) + X_O$ for the second, such that the estimated speeds on each leg are equal, i.e.

$$\left\| \mu(\hat{V}_1 - V_O) + V_O \right\| = \left\| \mu(\hat{V}_2 - V_O) + V_O \right\| \qquad (6)$$

with $\hat{V}_1 = \left[\hat{\dot{x}}_1 \quad \hat{\dot{y}}_1\right]^T$ and $\hat{V}_2 = \left[\hat{\dot{x}}_2 \quad \hat{\dot{y}}_2\right]^T$.

From (6), we deduce

$$\tilde{\mu} = -\frac{2(\hat{V}_1 - \hat{V}_2)^T V_O}{\left(\|\hat{V}_1\|^2 - \|\hat{V}_2\|^2 - 2(\hat{V}_1 - \hat{V}_2)^T V_O\right)} \qquad (7)$$

Then we can estimate new $\tilde{X}_{S,1} = \tilde{\mu}(\hat{X}_1 - X_0) + X_0$ and $\tilde{X}_{S,2} = \tilde{\mu}(\hat{X}_2 - X_0) + X_0$ and hence the corresponding leg-by-leg BOMTMA solution denoted $\tilde{Z}$.

Critical comments about the geometric fusion must be made here because

1) There is no guarantee that $\tilde{\mu} > 0$.
2) The covariance matrices of $\hat{X}_1$ and $\hat{X}_2$ are not exploited.
3) The probability that the second components of $\tilde{X}_{S,1}$ and $\tilde{X}_{S,2}$ are equal (the estimated trajectory is then discontinuous) is equal to 0 !

This motivates the statistical fusion presented hereafter.

4.2 The statistical fusion

Only step 2 changes.

The first step provided us $\hat{X}_1 = \left[\bar{\rho}\sin\beta_M + x_O(t_M) \quad \hat{y}_1 \quad \hat{\dot{x}}_1 \quad \hat{\dot{y}}_1\right]^T$ and $\hat{X}_2 = \left[\bar{\rho}\sin\beta_M + x_O(t_M) \quad \hat{y}_2 \quad \hat{\dot{x}}_2 \quad \hat{\dot{y}}_2\right]^T$ by the partial BOTMA from the bearings collected during $[t_1 \quad t_M]$ and $[t_M \quad t_K]$. Because we fixed the first component of $\hat{X}_1$ and $\hat{X}_2$, $\hat{\breve{X}}_1 = \left[\hat{y}_1 \quad \hat{\dot{x}}_1 \quad \hat{\dot{y}}_1\right]^T$ is the MLE of $\lambda(\breve{X}_{S,1} - \breve{X}_O) + \breve{X}_O$, and $\hat{\breve{X}}_2 = \left[\hat{y}_2 \quad \hat{\dot{x}}_2 \quad \hat{\dot{y}}_2\right]^T$ the MLE of $\lambda(\breve{X}_{S,2} - \breve{X}_O) + \breve{X}_O$, with $\lambda = \dfrac{\bar{\rho}\sin\beta_M}{x_S(t_M) - x_O(t_M)}$, $\breve{X}_O = [y_O(t_M) \quad \dot{x}_O \quad \dot{y}_O]^T$ and $\breve{X}_{S,i} = [y_S \quad \dot{x}_{S,i} \quad \dot{y}_{S,i}]^T$, $i = 1, 2$. In short, we have to estimate three unknowns: λ, $\breve{X}_{S,1}$ and $\breve{X}_{S,2}$. Once estimated, they allow us to estimate Z since it can be recovered from $\breve{X}_1$, $\breve{X}_2$ and $x_S(t_M)$ deduced from λ.

The key of the so called statistical fusion is to use $\hat{\breve{X}}_1$ and $\hat{\breve{X}}_2$ as measurements of $\lambda(\breve{X}_{S,1} - \breve{X}_O) + \breve{X}_O$ and $\lambda(\breve{X}_{S,2} - \breve{X}_O) + \breve{X}_O$.

Indeed, $\hat{\breve{X}}_i = \lambda(\breve{X}_{S,i} - \breve{X}_O) + \breve{X}_O + \breve{\varepsilon}_i$, for $i = 1,2$, with $\breve{\varepsilon}_i \sim G(\vec{0}, F^{-1}(\breve{X}_i))$, $F(\breve{X}_i)$ being the Fisher information matrix (FIM) of $\breve{X}_i$.

The statistical fusion then consists of minimizing the quadratic criterion:

$$C(Z) = \frac{1}{2}\left\| \hat{\breve{X}}_1 - \breve{X}_O - \lambda(\breve{X}_{S,1} - \breve{X}_O) \right\|^2_{F(\breve{X}_1)} + \frac{1}{2}\left\| \hat{\breve{X}}_2 - \breve{X}_O - \lambda(\breve{X}_{S,2} - \breve{X}_O) \right\|^2_{F(\breve{X}_2)} \qquad (8)$$

according to the classic notation $\|x\|^2_S = x^T S x$ (with S a definite positive matrix).

Remarks: In practice, $F(\breve{X}_i)$, which is unknown, must be replaced by $F\left(\hat{\breve{X}}_i\right)$ returned by the two partial BOTMA as suggested in [9].

The minimization is completed by the Gauss-Newton algorithm and the estimate is denoted $\hat{Z}$.

4.3 Extension: the trajectory of the source is composed of three or more legs

The trajectory of S is now composed of three legs at constant speed v_S, defined by two times of maneuver t_{M1} and t_{M2} (assumed known), the final time being t_K. If we choose as reference time $t^* = t_{M2}$, then the whole trajectory is characterized by the state vector

$Z = \left[x_S(t_{M2}) \quad y_S(t_{M2}) \quad v_S \quad h_{S,1} \quad h_{S,2} \quad h_{S,3}\right]^T$, $h_{S,3}$ being the heading of the source during the third leg. We assume that there exists at least $i \in \{1, 2\}$ such that $V_O^T\left(V_{S,i} - V_{S,i+1}\right) \neq 0$ (observability condition).

Step 1 (partial BOTMA on each leg): We perform a partial BOTMA, the reference times being t_{M1} for the first and the second leg, and t_{M2} for the third one. As in the case of two legs, we fix the first component of X_i, $i \in \{1, 2\}$ to the value $x'_{fix} = \bar{\rho}\sin\beta_{M1} + x_O(t_{M1})$, corresponding to $\lambda = \dfrac{\bar{\rho}\sin\beta_{M1}}{x_S(t_{M1}) - x_O(t_{M1})}$. We introduce for the third leg the vector $X_3 = \left[x''_{fix} \quad y_3 \quad \dot{x}_3 \quad \dot{y}_3\right]^T$, with $x''_{fix} = \bar{\rho}\sin\beta_{M2} + x_O(t_{M2})$ corresponding to $\lambda = \dfrac{\bar{\rho}\sin\beta_{M2}}{x_S(t_{M2}) - x_O(t_{M2})}$.

Consequently, we get three estimates denoted $\hat{\breve{X}}_1$, $\hat{\breve{X}}_2$ and $\hat{\breve{X}}_3$ satisfying the following equations

$$\hat{\breve{X}}_1 = \lambda_1\left(\breve{X}_{S,1} - \breve{X}_{O1}\right) + \breve{X}_{O1} + \breve{\varepsilon}_1$$
$$\hat{\breve{X}}_2 = \lambda_2\left(\breve{X}_{S,2} - \breve{X}_{O2}\right) + \breve{X}_{O2} + \breve{\varepsilon}_2$$
$$\hat{\breve{X}}_3 = \lambda_3\left(\breve{X}_{S,3} - \breve{X}_{O3}\right) + \breve{X}_{O3} + \breve{\varepsilon}_3$$

with $\lambda_1 = \lambda_2 = \dfrac{\bar{\rho}\sin\beta_{M1}}{x_S(t_{M1}) - x_O(t_{M1})}$,

$$\lambda_3 = \frac{\bar{\rho}\sin\beta_{M2}}{x_S(t_{M2}) - x_O(t_{M2})},$$

$\breve{X}_{O1} = \breve{X}_{O2} = \left[y_O(t_{M1}) \quad \dot{x}_O \quad \dot{y}_O\right]^T$ and

$$\breve{X}_{O3} = \left[y_O(t_{M2}) \quad \dot{x}_O \quad \dot{y}_O\right]^T.$$

Step 2 (statistical fusion): The statistical fusion consists of minimizing the quadratic criterion:

$$C(Z) = \frac{1}{2}\left\|\hat{\breve{X}}_1 - \breve{X}_{O1} - \lambda_1\left(\breve{X}_{S,1} - \breve{X}_{O1}\right)\right\|^2_{F(\breve{X}_1)} + \frac{1}{2}\left\|\hat{\breve{X}}_2 - \breve{X}_{O2} - \lambda_2\left(\breve{X}_{S,2} - \breve{X}_{O2}\right)\right\|^2_{F(\breve{X}_2)} + \frac{1}{2}\left\|\hat{\breve{X}}_3 - \breve{X}_{O3} - \lambda_3\left(\breve{X}_{S,3} - \breve{X}_{O3}\right)\right\|^2_{F(\breve{X}_3)}$$

Again, we replace the unknown FIM $F(\breve{X}_i)$ by their respective estimates $F\left(\hat{\breve{X}}_i\right)$, for $i \in \{1, 2, 3\}$.

5 Monte Carlo simulations

In all our simulations, we choose a sampling time Δt equal to 4 seconds and a standard deviation of the measurement noise equal to 1 degree. In Fig. 2, 3, 4, and 5, the letters S and O are located at the initial positions of the source and the observer, respectively.

5.1 Two legs: Comparison of the respective performance of the geometric and statistical fusion

We present two scenarios for a 500-run Monte Carlo simulation, named scenario #1 and scenario #2.

For both, the total duration of the scenario is 30 min. corresponding to 450 measurements. The motion of the observer is characterized by its speed of 6 m/s, with heading of 90° and its location $[9 \text{ km}, 0 \text{ km}]^T$ at time $t_M = 20\,\text{min}$. (known).

Scenario #1 is depicted in Fig. 2: The source has a speed of 5 m/s, with heading 108°, changes its course at t_M and its new heading is 200°; at this time, its location is $[5 \text{ km}, 10 \text{ km}]^T$. In Fig. 2a (respectively Fig. 2b), the 500 estimates given by the geometric (respectively statistical) fusion are plotted. The average of the coordinates of $\tilde{Z}$ and of $\hat{Z}$, their bias, their empirical standard deviations and the minimum standard deviations deduced from the Cramer Rao lower bound (CRLB) are given in Table 1.

For this first scenario, the Gauss-Newton method (for the statistical fusion) initialized by $\tilde{Z}$ (given by the geometric fusion) behaves properly (it never stalls on a local minimum) and reaches estimates whose performance is close to the ultimate one given by the CRLB.

Fig. 3 shows scenario #2, together with the results of 500 estimates given by the geometric (respectively statistical) fusion. The source has a speed of 5 m/s and a heading of 333°; its second heading is 18°. At time t_M , its location is $[-5 \text{ km}, 0 \text{ km}]^T$. For this scenario, the estimate $\tilde{\mu}$ is sometime negative yielding a unacceptable estimate $\tilde{Z}$. So, the Gauss Newton procedure, used for the statistical fusion, is initialized as follows:

$Z_{init} = \left[x_{init} \quad y_{init} \quad v_{init} \quad h_{1,init} \quad h_{2,init}\right]^T$ with,

$$x_{init} = \bar{\rho}\sin\beta_M + x_O(t_M),$$
$$y_{init} = \bar{\rho}\cos\beta_M + y_O(t_M),\ \bar{\rho} = 50\,km$$
$$v_{init} = \frac{1}{2}\left(\left\|V_{1,init}\right\| + \left\|V_{2,init}\right\|\right),$$
$$h_{j,init} = \tan^{-1}\left(\frac{\dot{x}_{j,init}}{\dot{y}_{j,init}}\right) \text{ for } j = 1, 2,$$
$$V_{1,init} = \begin{bmatrix}\dot{x}_{1,init}\\ \dot{y}_{1,init}\end{bmatrix} = \begin{bmatrix}\dfrac{\bar{\rho}\sin\beta_M + x_O(t_M) - \bar{\rho}\sin\beta_1 - x_O(t_1)}{t_M - t_1}\\ \dfrac{\bar{\rho}\cos\beta_M + y_O(t_M) - \bar{\rho}\cos\beta_1 - y_O(t_1)}{t_M - t_1}\end{bmatrix}$$

$$V_{2,init} = \begin{bmatrix} \dot{x}_{2,init} \\ \dot{y}_{2,init} \end{bmatrix} = \begin{bmatrix} \dfrac{\bar{\rho}\sin\beta_M + x_O(t_M) - \bar{\rho}\sin\beta_K - x_O(t_K)}{t_M - t_K} \\ \dfrac{\bar{\rho}\cos\beta_M + y_O(t_M) - \bar{\rho}\cos\beta_K - y_O(t_K)}{t_M - t_K} \end{bmatrix}$$

Again, the average of the coordinates of $\tilde{Z}$ and of $\hat{Z}$, their bias, their empirical standard deviation and the minimum standard deviations deduced from the CRLB are given in Table 2.

Note that in Fig. 2 and Fig. 3, the 90%-ellipsoids are plotted at time t_M and t_K.

5.2 Three legs

We added a third leg to the two previous trajectories of the source: scenario #3 (respectively scenario #4) has been constructed with scenario #1 (respectively scenario #2) by adding a third leg to the trajectory of the source corresponding to a new heading of 240° (respectively 50°). For both, the total duration of the scenario is 46 min. and 40 s. corresponding to 700 measurements. Fig. 4 (respectively 5) depicts an example of a 500-run Monte Carlo simulation for scenario #3 (respectively scenario #4). The initialisation is similar to the one used for scenario #2.

Scenario # 3: The average of the coordinates of $\hat{Z}$, the bias and the empirical standard deviation are given in Table 3. For this scenario, we do not observe any difficulty with the initialization and the Newton scheme performs.

Scenario #4: Table 4 summarizes the performance of the statistical fusion. The numerical method reveals its sensitivity to the initial point for this scenario.

Remark: we did not compute the CRLB (which poses any problem).

6 Conclusion

We have proposed an estimate for a two-leg source trajectory from bearings-only collected by a non maneuvering observer, based upon the statistical fusion of the MLE given by the partial BOTMA run on each leg. Moreover, the principle of estimating can be easily extended to the case of a multi-leg trajectory. The interest of this method is that we compute leg after leg a partial BOTMA which allows us to provide a quasi-efficient estimate at the end of each leg by statistical fusion. Nevertheless, the Gauss-Newton scheme used to reach the estimates suffers from its sensitivity to the initial point.

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Table 1: respective performance at the final time t_K of geometric ($\tilde{Z}$) and statistical ($\hat{Z}$) fusion for scenario #1 (see Fig. 2); boldface types are relative to the new estimate.

Z_K	Units	$Z_{K,True}$	$\tilde{Z}_{K,average}$	$\hat{Z}_{K,average}$	Bias($\tilde{Z}$)	Bias($\hat{Z}$)	σ_{CRLB}	$\tilde{\sigma}$	$\hat{\sigma}$
$x_S(t_K)$	km	3.974	3.946	**3.932**	0.028	**0.042**	0.528	0.748	**0.561**
$y_S(t_K)$	km	7.181	7.206	**7.219**	0.025	**0.038**	0.478	0.577	**0.504**
v_S	m/s	5	5.1	**5.1**	0.1	**0.1**	0.3	0.4	**0.3**
$h_{S,1}$	degrees	108	107.7	**107.8**	0.3	**0.2**	7.9	8.4	**7.4**
$h_{S,2}$	degrees	200	200.6	**200.5**	0.6	**0.5**	7.3	9.7	**7.4**

Table 2: respective performance at the final time t_K of geometric ($\tilde{Z}$) and statistical ($\hat{Z}$) fusion, for scenario #2 (see Fig. 3); boldface types are relative to the new estimate.

Z_K	Units	$Z_{K,True}$	$\tilde{Z}_{K,average}$	$\hat{Z}_{K,average}$	Bias($\tilde{Z}$).	Bias($\hat{Z}$)	σ_{CRLB}	$\tilde{\sigma}$	$\hat{\sigma}$
$x_S(t_K)$	km	-4.072	-6.235	**-5.902**	2.163	**1.83**	0.418	19.767	**3.44**
$y_S(t_K)$	km	2.853	3.212	**3.168**	0.359	**0.315**	0.118	3.357	**0.559**
v_S	m/s	5	6.7	**6**	1.7	**1**	0.7	9.7	**1.5**
$h_{S,1}$	degrees	333	336.6	**328.15**	3.54	**4.85**	5.24	29	**6**
$h_{S,2}$	degrees	18	21	**15.64**	2.97	**2.36**	21	28.5	**22**

Table 3: performance at the final time t_K, for scenario #3, (see Fig. 4)

Z_K	Units	$Z_{K,True}$	$\hat{Z}_{K,average}$	Bias	$\hat{\sigma}$
$x_S(t_K)$	km	-0.356	-0.335	0.021	1.303
$y_S(t_K)$	km	4.680	4.669	0.011	0.351
v_S	m/s	5	5.05	0.05	0.38
$h_{S,1}$	degrees	108	107.4	0.6	8.3
$h_{S,2}$	degrees	200	200.5	0.5	3.2
$h_{S,3}$	degrees	240	239.8	0.2	2.5

Table 4: performance at the final time t_K, for scenario #4, (see Fig. 5)

Z_K	Units	$Z_{K,True}$	$\hat{Z}_{K,average}$	Bias	$\hat{\sigma}$
$x_S(t_K)$	km	-0.243	-0.284	0.041	1.056
$y_S(t_K)$	km	6.067	6.101	0.034	0.337
v_S	m/s	5	5.5	0.5	0.6
$h_{S,1}$	degrees	333	331.4	2.6	3.8
$h_{S,2}$	degrees	18	19.3	1.3	15
$h_{S,3}$	degrees	50	53.8	3.8	7.8

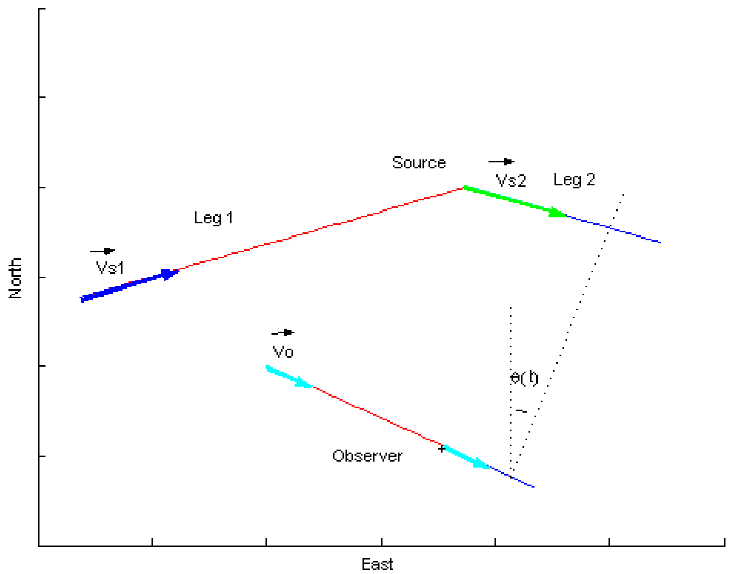

Fig. 1: Example of observer and source trajectories composed of two legs

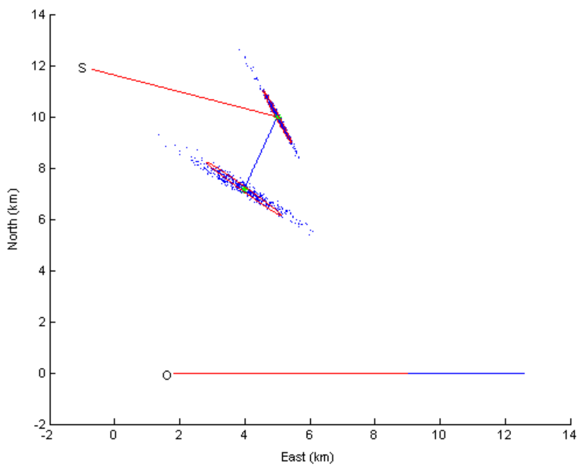

Fig. 2a: result of a 500-run simulation of the geometric fusion for scenario #1 ($\tilde{Z}$). The estimates and the corresponding 90%-ellipsoids (deduced from the CRLB) are plotted at the time of maneuver and at the final time.

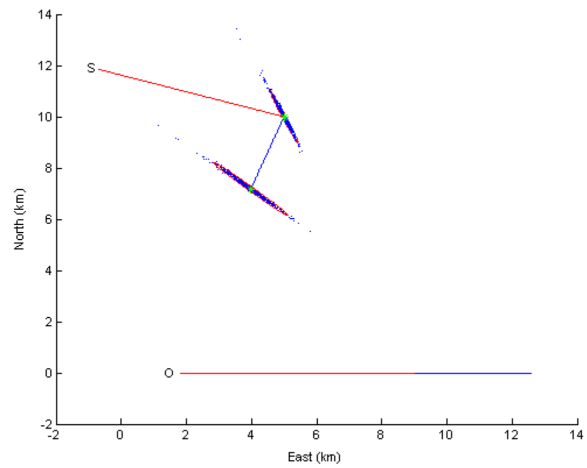

Fig. 2b: result of a 500-run simulation of the statistical fusion for scenario #1 ($\hat{Z}$).The estimates and the corresponding 90%-ellipsoids (deduced from the CRLB) are plotted at the time of maneuver and at the final time.

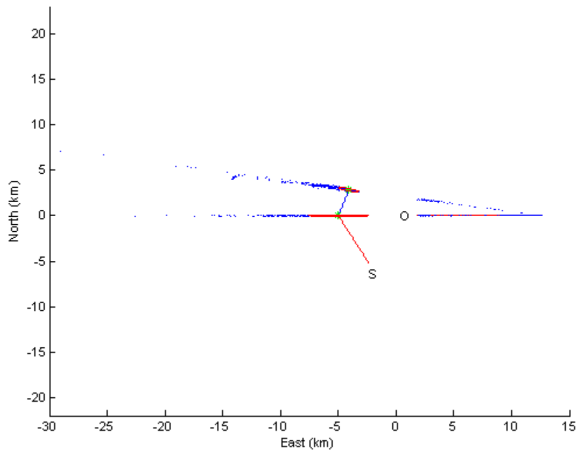

Fig. 3a: result of a 500-run simulation of the geometric fusion ($\tilde{Z}$) for scenario #2. The estimates and the corresponding 90%-ellipsoids (deduced from the CRLB) are plotted at the time of maneuver and at the final time.

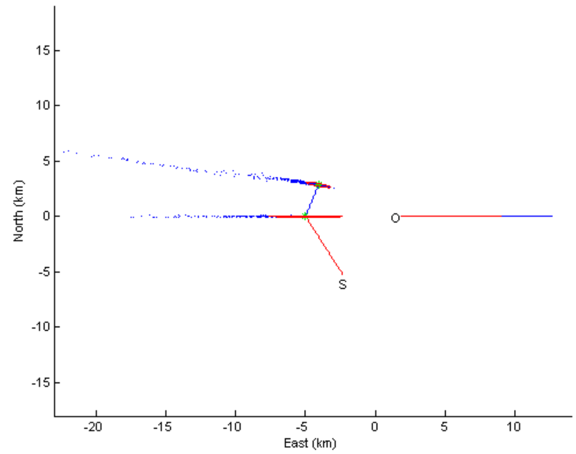

Fig.3b: result of a 500-run simulation of the statistical fusion ($\hat{Z}$) for scenario #2. The estimates and the corresponding 90%-ellipsoids (deduced from the CRLB) are plotted at the time of maneuver and at the final time.

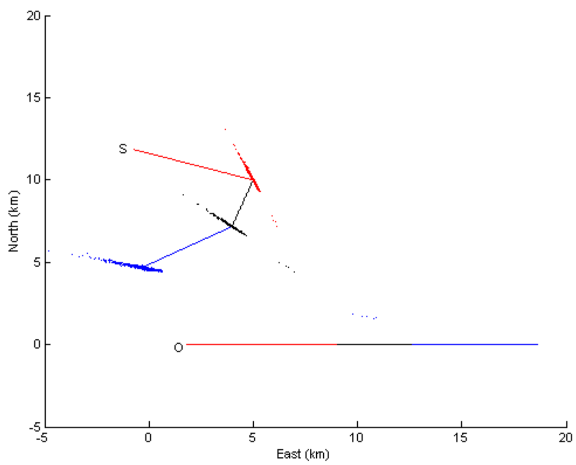

Fig. 4: result of a 500-run simulation of the statistical fusion ($\hat{Z}$) for scenario #3. The estimates are plotted at the times of maneuver and at the final time.

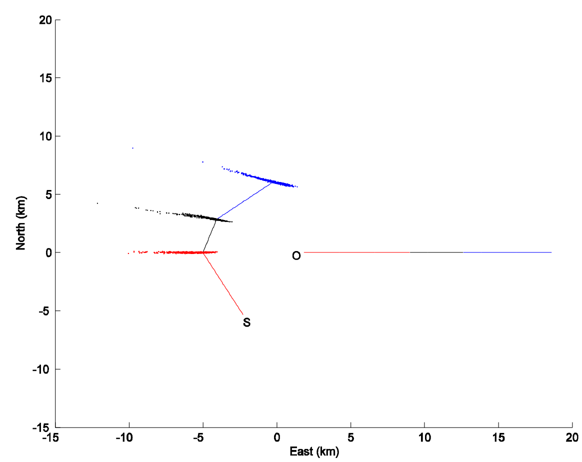

Fig. 5: result of a 500-run simulation of the statistical fusion ($\hat{Z}$) for scenario #4. The estimates are plotted at the times of maneuver and at the final time.